Uterine contractility signals – an introduction to wavelet analysis

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Abstract

Purpose: Analysis of the uterine contractility in the nonpregnant states has provided information about physiological changes during menstrual cycle. There is need to develop methods of recording uterine activity as well as mathematical interpretation of recorded time series. Wavelets are a new powerful tool for signal and image processing. The aim of this study is an introductory view of Fourier (one of the fundamental methods of investigating of biomedical signals) and wavelet transforms applications in the analysis of uterine contractions.

Material and methods: Spontaneous uterine activity of healthy patient and patient with dysmenorrhea was recorded by micro-tip two sensors catheter (Millar Instruments, Inc. USA). After amplification analogue signals were converted to digital. Signals were analysed using Fourier and wavelet transforms.

Results: Contrary to the Fourier decomposition, which is global and provides the information integrated over the whole signal, the continuous and discrete wavelet transforms allow to extract local and global variations of the recorded contractions. From the analysis of the coefficients of the wavelet transform we can assess various pattern of propagation: normal propagation, simultaneous propagation and inverted propagation.

Conclusions: This study is the introduction to the wavelet analysis of the uterine contraction signals. Wavelet transform provides insight into the structure of the time series at various scales. It allows to localise changes of the signal in time, providing additional information in comparison with the Fourier transform.

Key words: uterine contractions, Fourier transform, wavelet transform.

Introduction

Analysis of the uterine contractility in the nonpregnant states has provided information about physiological changes during the menstrual cycle [1]. There is need to develop methods of recording uterine activity as well as mathematical interpretation of recorded time series [2]. One of the fundamental methods of investigating various kinds of biomedical signals is the Fourier transform [3], which analyses signal in terms of periodic basis function (sine and cosine). Wavelets are a new powerful tool for signal and image processing. In the wavelet transform signal is decomposed into elementary components well localised in the time domain and in the frequency domain. This method has been successfully used in a number of fields. The wavelet transform have been used to analyse heart rate variability [4], EEG signals [5], electromyographic signals [6] and other kinds of biomedical signals [7].

The aim of this study is an introductory view of Fourier and wavelet transforms applications in the analysis of uterine contractions.

Data acquisition

Spontaneous uterine activity was recorded by a micro-tip catheter (Millar Instruments, Inc, USA). The study was approved by the regional ethics committee. The device consisted of two miniature pressure sensors (the distal sensor and the proximal sensor). The distance between sensors was 30 mm. The sensors produced electrical signals which varied in direct proportion to the magnitude of sensed pressures. After amplification, analogue signals were passed to IBM computer for conversion to digital form by means of an analogue-digital

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Figure 1. Fourier analysis of normal uterine contractions.

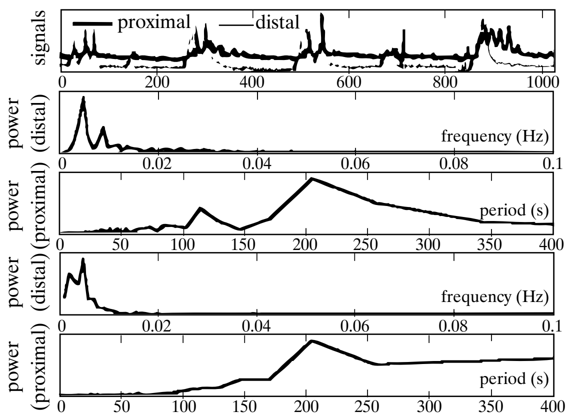
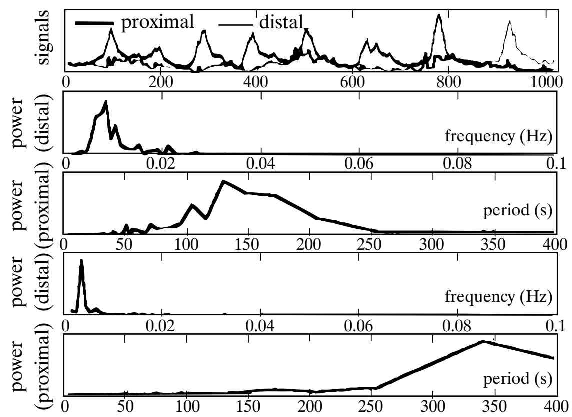


Figure 2. Fourier analysis of the contractions of the patient with dysmenorrhea.



(A/D) converter. Converted signals were recorded with a frequency 2 Hz on a computer hard disk. The sampling frequency may be changed in acquisition procedures. For analysis of the recorded signals programs written in MATLAB (MathWorks, Inc, USA), a high-performance language for technical computing, were used. Signal Processing Toolbox and Wavelet Toolbox appeared very helpful.

Fourier analysis

Intrauterine pressure signals may be analysed in the time domain or in the frequency domain. Parameters in the time domain such as area under curve recording (AUC), maximal amplitude of contractions and various statistical quantities (mean, standard deviation, median, skewness and so on) are easily computed even for short time window [7]. In frequency domain signal is decomposed by means of spectral analysis into its sinusoidal components. The Fourier transform (FT) of a signal x(t) is defined as [2]

$$X(f)=\int_{-\infty}^{+\infty} x(t)\exp(-2\pi ift)dt, \qquad (1)$$

where f is the frequency in cycles per unit time, usually in cycles per second (Hz).

The power spectrum or power spectral density function is given by

$$P(f)=|X(f)|^2. \qquad (2)$$

We can plot the power as a function of frequency. This information (frequency-based distribution of power) is very important from the medical diagnostics point of view. Fourier transform has some drawback. Frequencies that occur in one part of signal may be quite different from frequencies in the other part of it. There is no information about the localisation of the individual frequency components. Power spectrum gives only the frequency composition integrated over the whole signal or the whole analysing segment of the signal.

Basis function

$$\exp(-2\pi ift)=\cos(2\pi ift)-i\sin(2\pi ift), \qquad (3)$$

are perfectly localised in frequency domain, but they are not localised in time domain, they are infinite in time. *Fig. 1* shows an example of Fourier spectral analysis of uterine activity signals of normal patient performed by means of fast Fourier transform (FFT). For convenience, there are also the power versus period plot. Power spectra for distal and proximal signals are very similar. In *Fig. 2* there are the results of Fourier decompositions of the distal and the proximal signals of patient with dysmenorrhea. The power spectra for distal and proximal uterine activities are quite different (*Fig. 1*).

The Fourier transform is a global one. We have no information about localisation of the frequencies in time. Such information gives us the windowed Fourier transform (WFT), also known as short time Fourier transform (STFT) [9].

$$X(f,\tau)=\int_{-\infty}^{+\infty} x(t)w(t-\tau)\exp(-2\pi ift)dt, \qquad (4)$$

$X(f,\tau)$ is a two dimensional function of time and frequency, is the window outside of which the signal is suppressed (*Fig. 2*).

The spectrogram SPEC is defined as the square modulus of STFT

$$SPEC(f,\tau)=|X(f,\tau)|^2. \qquad (5)$$

$SPEC(f,\tau)$ is the energetic version of short time Fourier transform, it provides the energy distribution on a time-frequency plane. Often spectrogram is defined as the magnitude of STFT. Time-frequency localisation obtained by means of STFT is not precise. The product of the frequency and the time resolutions is constant

$$\Delta f\cdot\Delta t=const, \qquad (6)$$

so the increase of resolution in one domain causes the decrease of resolution in the other one. In uterine contraction activity signals low frequencies play the dominant role, thus the resolution in time domain is poor. Biomedical signals are in general very complicated. Sharp spikes, noise, non-stationarity result in Fourier decomposition of such time series into harmonic function being not satisfactory. *Fig. 3* shows that it is very difficult

Figure 3. The uterine contraction signal and its spectrogram.

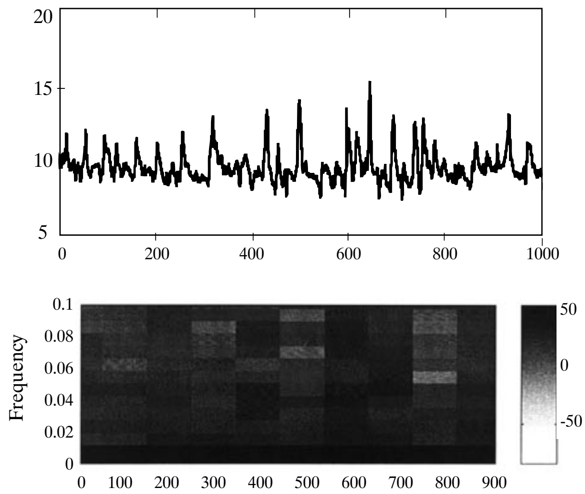
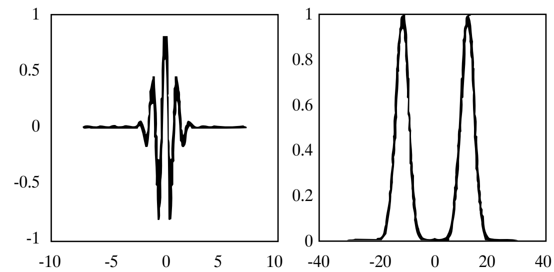


Figure 4. The Morlet wavelet $\psi(t)$ and the magnitude of the Fourier transform $\psi(t)$ of that wavelet.



to extract diagnostic information from spectrogram of uterine contraction activity signal (*Fig. 3*).

Wavelet transform

The wavelet transform is a new mathematical tool for the analysis of signals and images. The definition of the wavelet transform is similar to that of the Fourier transform. Instead of the periodic functions (sine and cosine) we use wavelets. The continuous wavelet transform (CWT) of a signal x(t) is given as [10]

$$CWT[x(t)]=W(a,b)=\int_{-\infty}^{+\infty}x(t)\psi_{a,b}(t)dt=\int_{-\infty}^{+\infty}x(t)\frac{1}{\sqrt{a}}\psi\left(\frac{x-b}{a}\right)dt \qquad (7)$$

where: $\psi_{a,b}(t)=\frac{1}{\sqrt{a}}\psi\left(\frac{x-b}{a}\right)$ is the analysing wavelet (mother wavelet),

a – determines dilation, b – specifies translation and $(-\infty<a, b<\infty, a\neq 0)$.

In other words 'a' means scale and 'b' means position. Thus the equation (1) may be rewritten in a simpler form

$$W(a,b)=W(\text{scale, position})=\int_{-\infty}^{+\infty}x(t)\psi(\text{scale, position})dt \qquad (8)$$

The term scale, used in wavelet analysis, is similar to the scale used in geographical maps. We can obtain small scales (high frequencies) or large scales (low frequencies) components of the analysed time series. In the wavelet transform we have an infinitive set of possible basis functions. Among them are Daubechies wavelets, Mexican hat wavelet, Meyer wavelet, Haar wavelet, Morlet wavelet, Coiflet wavelet, Spline wavelet and others. All of these functions are well localised in time and well localised in frequency. *Fig. 4* shows the Morlet wavelet and the magnitude of the Fourier transform of that wavelet $|\Psi(\omega)|$. We can see a good localisation of these functions in the time domain and in the frequency domain (*Fig. 4*).

For admissible mother wavelet (the Fourier transform of this function has neither a zero frequency component nor infinite frequency components) the equation (1) there is a convolution integral. Thus, wavelet analysis can be viewed as a filtering process. Coefficients W(a, b) are obtained by band--pass filtering signal x(t) by wavelet ψ. If wavelet coefficients are large the resemblance between the signal and the wavelet is strong, otherwise it is slight.

It is possible to analyse a signal in time-scale plane with relative accuracy. Time-frequency plane is equivalent to time--scale plane. The wavelet spectrogram which is also named wavelet scalogram or shorter scalogram is defined as [10,11].

$$WSCAL(a,b)=|W(a,b)|^2. \qquad (9)$$

Scalogram shows the distribution of energy in the time-scale plane. Scalogram can also be defined as a the magnitude of wavelet coefficients. Large scale corresponds to a stretched wavelet, small scale corresponds to a compressed wavelet (*Fig. 5*).

Fig. 5 shows us four scalograms of uterine contractions time series. We can see that for the normal contractions patterns the distal and the proximal signals are very similar. In the case of dysmenorrhea contractions, patterns are quite different. The local maxima of the absolute values of the wavelet coefficients reveal the occurrence of sharp time series variations. Local minima are connected with the occurrence of slow signal changes. The sharp temporal variations are zooming in over all scales.

Discrete wavelet transform

The disadvantages of the continuous wavelet transform are computational complexity and redundancy. They can be reduced by discretising scale 'a' and position 'b'. In dyadic representation, scale and position are based on power of two, so the wavelet decomposition works like a cascaded octave band-pass filter. Octave is the interval where the frequency at the end is twice the frequency at the beginning. For a forward discrete wavelet transform

$$DWT[x(t)]=W(j,k)=\sum_{n\in Z}x(t)\psi_{j,k}(t), \qquad (10)$$

the scale and position are based on power of two ($a=2^j$, $b=k2^j$ – dyadic representation), j, k are the level and position parameters and $\psi_{j,k}(t)=2^{-j/2}\,\psi(2^{-j}t-k)$.

Wavelet coefficients W (j,k) = W (level, position) at the

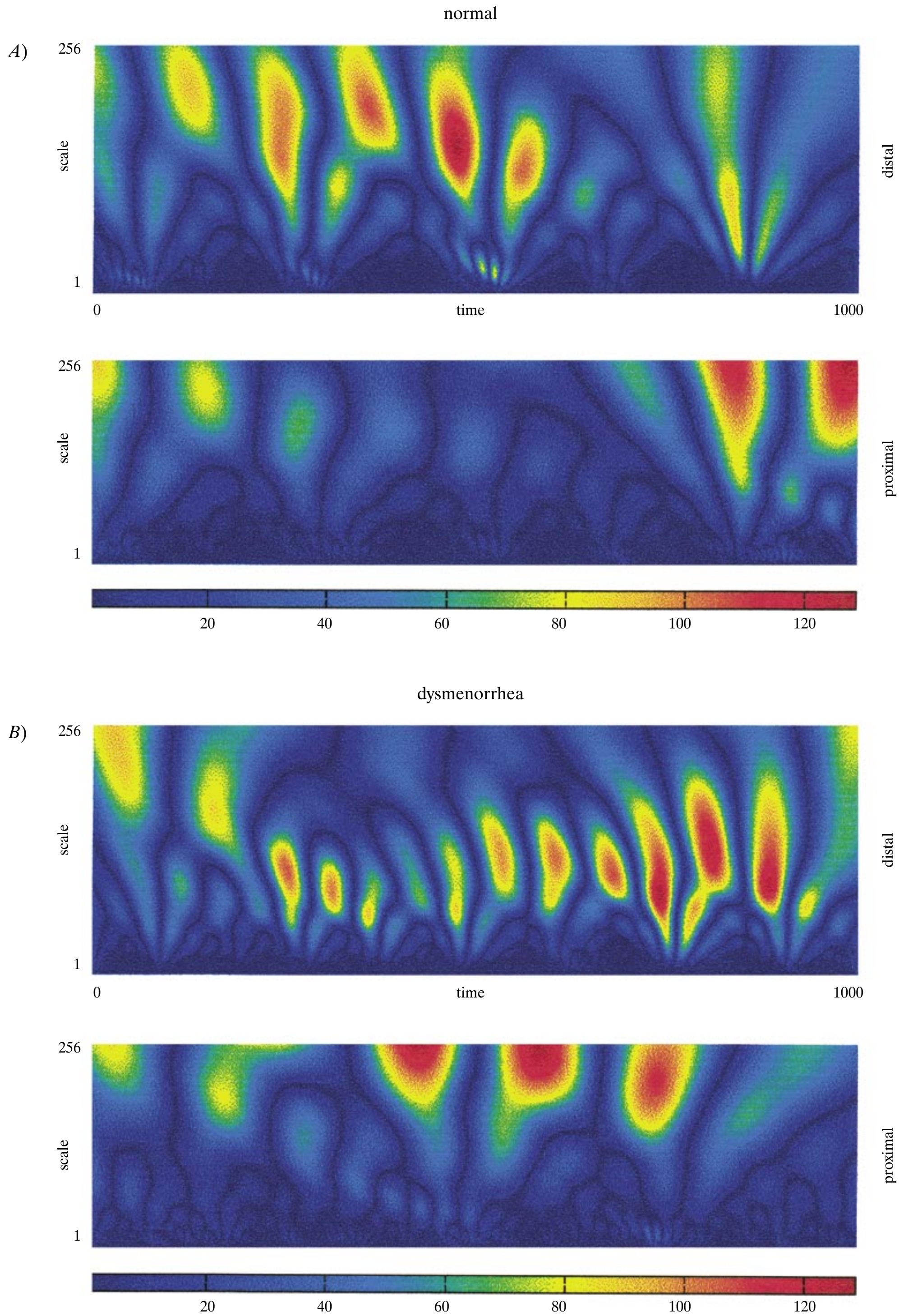


Figure 5. *A*) the scalograms of normal uterine contractions, *B*) the scalograms of contractions of patient with dysmenorrhea.

lower parameter j represent the characteristic of the signal at higher frequency and vice versa. In discrete wavelet decomposition, wavelet coefficients are computed for $j = 1, 2, 3, \ldots$. At each level j the number of W(j,k) is reduced by 2^j. In practise, the maximum of 11 steps of wavelet decomposition is sufficient.

An efficient way to compute the discrete wavelet transform is the multiresolution signal decomposition algorithm (MRSDA), known also as the Mallat algorithm [12]. In the multiresolution algorithm time series is decomposed into a collection of orthonormal function, which are obtained by transactions and dilations of a scaling function $\varphi(t)$ and the wavelet $\psi(t)$. In practise, here the signal is convolved with a pair of quadrature mirror decomposition filters (QMF's) – lowpass filter L and highpass filter H and afterwards results are downsampled by a factor of two. This operation splits the signal bandwidth in half. As the results approximation coef-

Figure 6. The discrete wavelet decomposition of a signal.

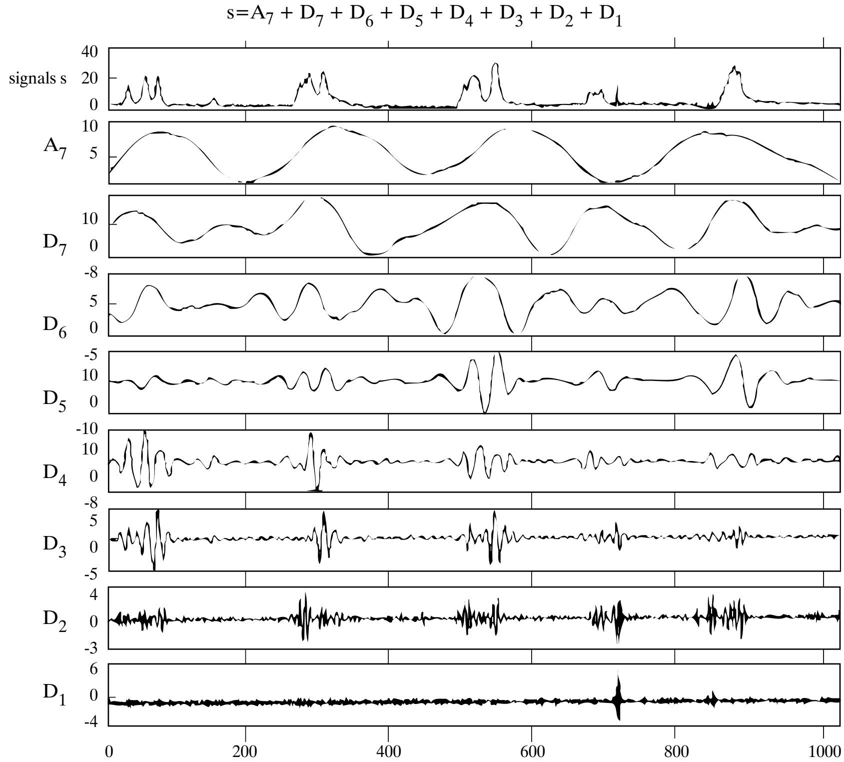
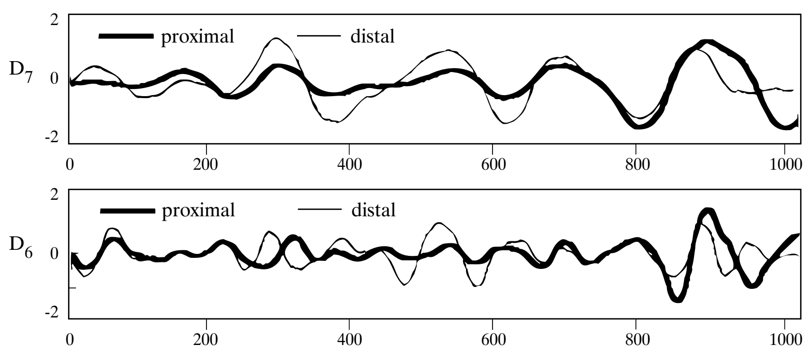


Figure 7. Assessing the propagation of the signal from details D_6 and D_7.



ficients cA_1 and detailed coefficients cD_1 are obtained. The process is iterated but only the approximation coefficients are further processed. From wavelet coefficient obtained by means of Mallat algorithm we can reconstruct approximations A_j and details D_j, so for a given reference level J, signal x(t) may be expressed as:

$$x(t)=A_J(t)=\sum_{j\leq J} D_j(t), \qquad (11)$$

Approximations are the high scale low frequency components of the signal. Details are low scale high frequency component of the signal.

Fig. 6 shows how the time series is built from its approximation and details components. The level of performed discrete wavelet transform was 7. We reconstructed approximation A_7 and details $D_j (1\leq j\leq 7)$. This seven wavelet decomposition represents the following frequency bands (from D_1 to D_7): 0.25-0.5 Hz; 0.125-0.25 Hz; 0.0625-0.125 Hz; 0.03125-0.0625 Hz; 0.015625-0.031125 Hz; 0.0078125-0.015625 Hz; 0.00390625--0.0078125 Hz. These are equivalent to the following period bands: 2-4 s; 4-8 s; 8-16 s; 16-32 s; 32-64 s; 64-128 s; 128-256 s. Approximation is the component between 0Hz and the lowest frequency at the frequency band at level 7 (*Fig. 6*).

Propagation of uterine contractions signals

By means of the wavelet transform we can determine the propagation of the uterine contraction signals [13]. From the details D_j we can assess the time lag between the distal and proximal signals. There are various patterns of propagation of contractions:

– normal propagation – the uterine fundus contracts before the uterine os, positive lag,

– inverted propagation – the uterine fundus contracts after the uterine os, negative lag,

– simultaneous propagation – the uterine fundus and the uterine os contract simultaneously, lag equals 0.

Fig. 7 shows that from the details D_6 and D_7 we can assess the direction of propagation of the signal. It can be seen that in both cases the uterine fundus contracts before the uterine os. The proximal signals are delayed in relation to distal signals (*Fig. 7*).

Concluding remarks

This study is the introduction to the wavelet analysis of the uterine contraction signals. We discuss both the Fourier transform and the wavelet transform. Wavelet transform provides insight into the structure of the time series at various scales. It allows to localise changes of the signal in time, providing additional information in comparison with the Fourier transform. Wavelet analysis may be used in the research on dysmenorrhea and endometriosis as an additional tool.

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